

Erratum

“Symmetry of stochastic non-variational differential equations”

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In my recent paper [1], due to a regrettable and rather trivial mistake, a mixed derivatives term is missing in the expression (5.3) for the Ito Laplacian – which is essentially a Taylor expansion. The correct formula is, of course

$$\Delta f := \sum_{k=1}^n \frac{\partial^2 f}{\partial w^k \partial w^k} + \sum_{j,k=1}^n (\sigma \sigma^T) \frac{\partial^2 f}{\partial x^j \partial x^k} + 2 \sum_{j,k=1}^n \sigma^{jk} \frac{\partial^2 f}{\partial x^j \partial w^k}. \quad (5.3)$$

(The reader is alerted that the same mistake found its origin in a previous paper of mine [2], on which some of this review was based.)

This error has no consequence on our general discussion – conducted in terms of the Δ operator – except for what is said below; but it does affect the specific computations occurring in most of the concrete examples of Section 5.

The error in (5.3) has some more substantial consequence in Remark 5.8 and Section 5.5.

The part of Remark 5.8 following eq.(5.32) is simply wrong: once the correct formula for $\Delta(\varphi)$ is used, the quantity δ^i defined in eq.(5.33) is exactly zero, in any dimension, as proved in [3].

All the discussion in Sect.5.5 should be revised in the light of this fact; in particular, $\delta^i = 0$ means that for *simple* (deterministic or random) *symmetries*, there is a full equivalence between an Ito and the corresponding Stratonovich equation. (In the deterministic case, this was proved by Unal, see ref.200 in [1].)

Note this holds in *any* dimension (while in Section 5.5 we only considered the scalar case); the correct statement concerning the matter considered in this Section is therefore as follows:

“The simple (deterministic or random) symmetries of an Ito equation and those of the corresponding Stratonovich one do coincide”.

I apologize to the readers, and thank the anonymous Referee of [3] for pointing out the mistake.

[1] G. Gaeta, “Symmetry of stochastic non-variational differential equations”, *Physics Reports* **686** (2017), 1-62

[2] G. Gaeta and F. Spadaro, “Random Lie-point symmetries of stochastic differential equations”, *J. Math. Phys.* **58** (2017), 053503

[3] G. Gaeta and C. Lunini, “On Lie-point symmetries for Ito stochastic differential equations”, to appear in *J. Nonlin. Math. Phys.*